Int J Theor Phys (2009) 48: 3049–3060
DOI 10.1007/s10773-009-0100-4

A New Class of Bianchi Type-I Cosmological Models in Lyra Geometry

G.P. Singh · A.Y. Kale

Received: 12 May 2009 / Accepted: 15 July 2009 / Published online: 28 July 2009

Abstract This paper deals with Bianchi Type-I cosmological models of the universe with uniform and variable gravitational constant and bulk viscosity in Lyra Geometry. A new class of exact solutions of field equations has been obtained. Physical behaviors of the models have also been discussed.

Keywords Lyra Geometry · Bianchi Type-I space time · Bulk viscosity · Shear scalar

1 Introduction

The Einstein's idea of geometrizing gravitational field inspired researchers to geometrize other fields of the physics. Einstein's general theory of relativity provides a geometrical description of gravitation and it is based on Riemannian geometry with metric tensor g_{ij}. In 1918 Weyl [1] presented a more general theory in order to geometrize gravitation and electromagnetism both. In Weyl's geometry, in addition to metric tensor g_{ij}, there is also a vector field φ_i such that under parallel displacement of a vector not only the direction but also the length may change and the change of length depends on φ_i. The idea of change of length of a vector under parallel displacement was criticized by Einstein because it implies that frequency of spectral lines emitted by atoms would not remain constant but would depend on their past histories which is in contradiction to the observed uniformity of their properties.

Lyra [2] suggested a modification of Riemannian geometry by introducing a gauge function into the structureless manifold which bears close resemblance to Weyl geometry. Sen [3] presented the static model in Lyra's geometry with finite density which is similar to

G.P. Singh (✉)
Department of Mathematics, Visvesvaraya National Institute of Technology, Nagpur, India
e-mail: gpsingh@mth.vnit.ac.in

A.Y. Kale
Department of Mathematics, St. Vincent Pallotti College of Engineering and Technology, Nagpur, India
e-mail: ashwini_kale@rediffmail.com

the static Einstein model, but significant difference was that the model exhibited redshift. Halford [4] studied a nonstatic cosmological model within the framework of Lyra's geometry. Further he has suggested that the constant vector displacement vector field φ_i in Lyra's geometry behaves like a cosmological constant Λ in Einstein's general theory of relativity. Sen and Dunn [5] have introduced a scalar-tensor theory within the framework of Lyra's geometry in which both the scalar and tensor fields have intrinsic geometrical significance. Subsequently Halford [6] had shown that the scalar-tensor theory based on Lyra's geometry presented the same effects, within observational limits, as the Einstein's theory. Bhamra [7], Karade and Borikar [8], Reddy and Innaiah [9] have studied various cosmological models in Lyra's geometry with a constant displacement field. However, this restriction of the displacement field to be constant is merely one of convenience and there is no a priori reason for it. Soleng [10] studied cosmological model in Lyra geometry and suggested that the cosmologies based on Lyra geometry will either include a creation field or a cosmological term. Beesham [11] considered FRW models with time dependent displacement field. He has shown that by assuming the energy density of the universe to be equal to its critical value, the models have $k = -1$ geometry.

It is well known that the exact solution of general theory of relativity for homogeneous space time belong to either Bianchi type or Kantowaski-Sachs space time. Singh and Singh [12–15] have presented Bianchi-I, III, V, VI_0 and Kantowaski Sachs cosmological model with time dependent displacement field and have made a comparative study of Roberson-Walker models with constant deceleration parameter in the presence of cosmological term in Einstein theory with the cosmological theory based on Lyra's geometry. They have presented a review of cosmological models in Lyra's geometry [16]. Singh and Desikan [17] studied FRW model with time dependent displacement field with constant deceleration parameter. Rahaman et al. [18, 19] have studied inhomogeneous spherically symmetric higher dimensional model in presence of a mass less scalar field and cosmological models with negative constant deceleration parameter in Lyra's geometry. Bali and Chandnani [20, 21] discussed Bianchi Type I cosmological model for perfect fluid distribution and Bianchi type-III bulk viscous dust filled cosmological model in Lyra geometry. Singh [22] has discussed Bianchi type-V anisotropic cosmological model in the presence and absence of a magnetic field within the framework of Lyra geometry. Rahaman et al. [23] have studied the gravitational field of domain wall in Lyra Geometry. Using Lyra geometry, Pradhan [24, 25] had presented cosmological models of plane symmetric inhomogeneous perfect fluid distribution with electromagnetic field and plane symmetric thick domain wall with bulk viscosity. Very recently Bali and Chandnani [26] have investigated Bianchi type-V barotropic perfect fluid cosmological model in Lyra geometry.

Dissipative effects, including both the bulk and shear viscosities, are supposed to play a very important role in the early evolution of the universe. From a physical point of view the inclusion of dissipative terms in the energy momentum tensor of the cosmological fluid seems to be the best motivated generalization of the matter term of the gravitational field equation. A number of authors Gron [27], Maartens [28], Mak and Harko [29], Singh and Beesham [30] have studied the effect of bulk viscosity on the evolution of the universe in different context. The Einstein's field equations with bulk viscosity and variable G and Λ for Bianchi type universes are studied by Pradhan and Pandey [31]. Mak and Harko [32] have considered the dynamics of a causal bulk viscous cosmological fluid with constantly decelerating Bianchi type-I space time. Wang [33], Yadav et al. [34], Calistete et al. [35] and others have discussed a variety of bulk viscous isotropic and anisotropic cosmological models. Very recently Singh and Kumar [36] presented Bianchi Type-I cosmological model in presence of a dissipative fluid.

Motivated by aforesaid research works, in this paper we have studied Bianchi type-I cosmological model with uniform and variable G and bulk viscosity in full causal theory within the frame work of Lyra geometry.

2 Field Equations

The field equations with normal gauge in Lyra's geometry, take the form

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\varphi_i\varphi_j - \frac{3}{4}g_{ij}\varphi_k\varphi^k = -8\pi G T_{ij}. \tag{1}$$

Here, φ_i is a displacement vector field defined as $\varphi_i = (0, 0, 0, \beta)$ where $\beta = \beta(t)$ and other symbols have their usual meaning as in Riemannian geometry.

The energy momentum tensor is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}. \tag{2}$$

Here ρ is the energy density, p represents equilibrium pressure and u_i is the flow vector satisfying the relation $u^i u_i = 1$.

3 The Metric and Field Equations

The Bianchi type I metric is given by

$$ds^2 = dt^2 - R_1^2(t)dx^2 - R_2^2(t)dy^2 - R_3^2(t)dz^2, \tag{3}$$

where $x^i = (x, y, z, t)$.

The field equations (1) for the space-time metric (3) yield following equations

$$\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} + \frac{\dot{R}_3}{R_3}\frac{\dot{R}_1}{R_1} = 8\pi G\rho + \frac{3}{4}\beta^2, \tag{4}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} = -8\pi G p - \frac{3}{4}\beta^2, \tag{5}$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} = -8\pi G p - \frac{3}{4}\beta^2, \tag{6}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_3}{R_3} = -8\pi G p - \frac{3}{4}\beta^2. \tag{7}$$

By combining (4)–(7) one can easily obtain continuity equation as

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) + \rho\frac{\dot{G}}{G} + \frac{1}{8\pi G}\left(\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left\{\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right\}\right) = 0. \tag{8}$$

The energy-momentum conservation equation ($T^{ij}_{;j} = 0$) suggests

$$\dot{\rho} + 3(\rho + p)H = 0. \tag{9}$$

From (8) and (9), we have

$$\rho\frac{\dot{G}}{G}+\frac{3}{16\pi G}\beta^2\left(\frac{\dot{\beta}}{\beta}+3H\right)=0, \tag{10}$$

where

$$H=\frac{1}{3}\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right).$$

The physical quantities expansion scalar θ, shear scalar σ^2, relative anisotropy and deceleration parameter q are important in the cosmology on observational point of view. These quantities are defined as

$$\theta=3H=\frac{\dot{V}}{V}, \tag{11}$$

$$\sigma^2=\frac{1}{12}\left[\left(\frac{\dot{R}_1}{R_1}-\frac{\dot{R}_2}{R_2}\right)^2+\left(\frac{\dot{R}_2}{R_2}-\frac{\dot{R}_3}{R_3}\right)^2+\left(\frac{\dot{R}_3}{R_3}-\frac{\dot{R}_1}{R_1}\right)^2\right], \tag{12}$$

$$\text{Relative anisotropy}=\frac{\sigma^2}{\rho} \quad \text{and} \tag{13}$$

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1, \tag{14}$$

where volume of Bianchi type I universe is defined as $V=R_1R_2R_3$.

4 Cosmological Solutions

It can be easily seen that we have four independent equations (4)–(7) with seven unknowns R_1, R_2, R_3, ρ, p, G and β. In order to obtain exact solution, we require three more physically plausible relations amongst the variables.

Using the relation $V=R_1R_2R_3$ one can easily obtain the following solutions of (5)–(7)

$$R_1=A_1V^{1/3}\exp\int\frac{c_1}{3V}dt, \tag{15}$$

$$R_2=A_2V^{1/3}\exp\int\frac{c_2}{3V}dt, \tag{16}$$

$$R_3=A_3V^{1/3}\exp\int\frac{c_3}{3V}dt. \tag{17}$$

Here A_1, A_2, A_3 and c_1, c_2, c_3 are constants of integration satisfying the relation $A_1.A_2.A_3=1$ and $c_1+c_2+c_3=0$.

4.1 Cosmological Models with Uniform Gravitational Constant

As a first step towards finding the solutions of Einstein's field equations, we consider uniform value of gravitational constant i.e.

$$G=K\ \text{(constant)}. \tag{18}$$

Under this assumption equation (10) reduces to

$$\frac{\dot{\beta}}{\beta} + 3H = 0, \tag{19}$$

which suggests $\beta \propto \frac{1}{V}$ i.e. the displacement field decreases with expansion of the universe.

Case I: Power Law relation between displacement field and cosmic time

It has already been mentioned in the literature that displacement vector field β in Lyra geometry behaves like cosmological constant in general relativity. A number of phenomenological cosmological constant models have been discussed by several authors [37]. The evolutionary relation for cosmological constant as $\Lambda \propto t^{-\alpha}$ is studied by Beesham [38] and Kalligas et al. [39, 40].

In this case we have assumed similar relations between displacement field β and cosmic time t as

$$\beta = \beta_0 t^{-n}. \tag{20}$$

By use of assumption (20), (19) yields

$$V = V_0 t^n. \tag{21}$$

Further, using the value of V in terms of cosmic time, we can easily obtained the following expression for scale factors

$$R_1 = A_1 V_0^{1/3} t^{n/3} \exp\left[\frac{c_1 t^{1-n}}{3V_0(1-n)}\right], \tag{22}$$

$$R_2 = A_2 V_0^{1/3} t^{n/3} \exp\left[\frac{c_2 t^{1-n}}{3V_0(1-n)}\right], \tag{23}$$

$$R_3 = A_3 V_0^{1/3} t^{n/3} \exp\left[\frac{c_3 t^{1-n}}{3V_0(1-n)}\right]. \tag{24}$$

Now with the help of (20), (22)–(24), (4) and (5) suggests

$$\rho = \left(\frac{k_0}{t^2} + \frac{k_1}{t^{2n}}\right), \tag{25}$$

$$p = -\left[\frac{l_0}{t^2} + \frac{l_1}{t^{2n}}\right], \tag{26}$$

where

$$k_0 = \frac{n^2}{24\pi K} \quad \text{and} \quad k_1 = \frac{1}{8\pi K}\left(\frac{c_1c_2 + c_2c_3 + c_1c_3}{9V_0^2} - \frac{3\beta_0^2}{4}\right),$$

$$l_0 = \frac{n(n-2)}{24\pi K}, \qquad l_1 = \frac{1}{8\pi K}\left(\frac{c_1^2 + c_2^2 + c_1c_2}{9V_0^2} + \frac{3\beta_0^2}{4}\right).$$

The physical quantities of observational interests have following expression in terms of t,

$$\theta = \frac{n}{t}, \tag{27}$$

$$\sigma^2 = \frac{(c_1^2 + c_2^2 + c_3^2)}{36 V_0^2 t^{2n}}, \tag{28}$$

$$\text{relative anisotropy} = \frac{c_1^2 + c_2^2 + c_3^2}{36 V_0^2 [k_0 t^{2n-2} + k_1]}, \tag{29}$$

$$q = -1 + \frac{3}{n}. \tag{30}$$

It can easily seen that energy density, pressure, expansion scalar, shear and relative anisotropy, are decreasing with evolution of the universe. The deceleration parameter suggests that for all $n > 3$ the model presents accelerating expansion of the universe. The anisotropy dies out and the model approaches to isotropic model with evolution of the universe.

Case II: Power Law relation between displacement field and Hubble parameter

Several authors [41–44] have considered a well established relation between cosmological constant and Hubble parameter H as $\Lambda = \Lambda_0 H^2$. Motivated by these study we consider the displacement vector field as

$$\beta = \beta_0 H^2. \tag{31}$$

Substituting the values of G and β from (18) and (31) into (10), we get

$$V = \frac{1}{4}(at + b)^2. \tag{32}$$

With the help of (32), (15)–(17) reduce to

$$R_1 = \frac{A_1}{\sqrt[3]{4}}(at + b)^{2/3} \exp\left[\frac{-4c_1}{3a(at + b)}\right], \tag{33}$$

$$R_2 = \frac{A_2}{\sqrt[3]{4}}(at + b)^{2/3} \exp\left[\frac{-4c_2}{3a(at + b)}\right], \tag{34}$$

$$R_3 = \frac{A_3}{\sqrt[3]{4}}(at + b)^{2/3} \exp\left[\frac{-4c_3}{3a(at + b)}\right]. \tag{35}$$

Further the energy density and pressure are obtained from (4) and (5) as

$$\rho = \left(\frac{k_2}{(at + b)^2} + \frac{k_3}{(at + b)^4}\right), \tag{36}$$

$$p = \frac{-l_2}{(at + b)^4}, \tag{37}$$

where

$$k_2 = \frac{a^2}{6\pi K} \quad \text{and} \quad k_3 = \frac{12(c_1 c_2 + c_2 c_3 + c_1 c_3) - \beta_0^2 a^4}{54\pi K},$$

$$l_2 = \frac{12(c_1^2 + c_2^2 + c_1 c_2) + \beta_0^2 a^4}{54\pi K}.$$

In this case the physical quantities of observational interests have following expression in terms of t,

$$\theta = \frac{2a}{(at+b)}, \tag{38}$$

$$\sigma^2 = \frac{4}{9(at+b)^4}[c_1^2 + c_2^2 + c_3^2], \tag{39}$$

$$\text{relative anisotropy} = \frac{4(c_1^2 + c_2^2 + c_3^2)}{9[k_2(at+b)^2 + k_3]}, \tag{40}$$

$$q = \frac{1}{2}. \tag{41}$$

In this model the value of deceleration parameter is positive which shows decelerating behaviour of the cosmological model. It is worthwhile to mention that the WMAP observations are also consistent with the decelerating model (please refer [45, 46] and reference there in). The shear vanishes and model approaches to isotropic model with expansion of the universe.

4.1.1 Cosmological models with uniform gravitational constant in the presence of bulk viscosity

The presence of bulk viscosity leads to inflationary like solutions in general relativity [47]. It has been stated in the literature that the conventional theory of the evolution of the universe includes a number of dissipative processes such as the decoupling of neutrions during radiation era, entropy production, string creation, GUT phase transition (refer Maartens [28] and references there in). Very recently Bianchi type I cosmological models in presence of variable Λ with gravitational constant and Bulk viscosity have been discussed in the reference [48] and [49] respectively. The inclusion of dissipative term in the energy momentum tensor of the cosmological field is widely accepted as generalization of matter term of the gravitational field.

The energy momentum tensor in the presence of viscosity takes the form

$$T_{ij} = (\rho + p + \Pi)u_i u_j - (p + \Pi)g_{ij}. \tag{42}$$

The Einstein's field equations (1) for the space-time metric (3) yield following equations

$$\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} + \frac{\dot{R}_3}{R_3}\frac{\dot{R}_1}{R_1} = 8\pi G\rho + \frac{3}{4}\beta^2, \tag{43}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} = -8\pi G(p + \Pi) - \frac{3}{4}\beta^2, \tag{44}$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} = -8\pi G(p + \Pi) - \frac{3}{4}\beta^2, \tag{45}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_3}{R_3} = -8\pi G(p + \Pi) - \frac{3}{4}\beta^2. \tag{46}$$

By combining (43)–(46) one can easily obtain the continuity equation as

$$\dot{\rho} + (\rho + p + \Pi)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) + \rho\frac{\dot{G}}{G} + \frac{1}{8\pi G}\left(\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left\{\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right\}\right) = 0. \tag{47}$$

Now the energy-momentum conservation equation ($T^{ij}_{;j}=0$) demands that

$$\dot{\rho}+3(\rho+p+\Pi)H=0. \tag{48}$$

From (38) and (39), we have

$$\rho\frac{\dot{G}}{G}+\frac{3}{16\pi G}\beta^2\left(\frac{\dot{\beta}}{\beta}+3H\right)=0. \tag{49}$$

Equation of state

$$p=\gamma\rho,\quad 0\le\gamma\le 1. \tag{50}$$

For the full causal non equilibrium thermodynamics the causal evolution equation for bulk viscosity is

$$\Pi+\tau\dot{\Pi}=-\xi\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)-\frac{\varepsilon\tau\Pi}{2}\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}+\frac{\dot{\tau}}{\tau}-\frac{\dot{\xi}}{\xi}-\frac{\dot{T}}{T}\right). \tag{51}$$

Here $T\ge 0$ is the absolute temperature, ξ is the bulk viscosity coefficient which cannot become negative otherwise the principle of entropy increase would be violated and the coefficient τ denotes the relaxation time for transient bulk viscous effects. When $\varepsilon=0$, (51) reduces to evolution equation for truncated theory. For full causal theory $\varepsilon=1$ and the non-causal theory (Eckart's theory) has $\tau=0$ [28]. Now we will discuss effect on bulk viscosity with evolution of universe in non-causal, truncated and full causal theory respectively.

On the basis of Gibb's integrability condition, Maartens [50] has pointed out that the equation of state for temperature suggests

$$T\propto\exp\int\frac{dp(\rho)}{\rho+p(\rho)}.$$

which with the help of (50) reduces to

$$T=T_0\rho^{\frac{\gamma}{(1+\gamma)}}, \tag{52}$$

where T_0 stands for a constant.

Further, in order to have exact solutions of the system of equations one more physical plausible relation is required. Thus, we consider the well accepted relation

$$\tau=\frac{\xi}{\rho}. \tag{53}$$

Case I: Power Law relation between displacement field and cosmic time.

Considering same form of the displacement vector field $\beta=\beta_0 t^{-n}$ as in previous section, we get the same value of V as in (21), using the value of V in terms of cosmic time, we can easily obtained the relations between scale factors and cosmic time t, as shown in (22)–(24).

Now, with the help of (22)–(24), (49) and (50), (43), (44) suggest

$$\rho=\left(\frac{k_0}{t^2}+\frac{k_1}{t^{2n}}\right), \tag{54}$$

Table 1 Forms of bulk viscosity coefficient when $\beta = \beta_0 t^{-n}$

Non causal theory ($\tau = 0$)	$\xi = [\frac{m_0}{nt} + \frac{m_1}{nt^{2n-1}}]$
Truncated causal theory ($\varepsilon = 0$)	$\xi = \frac{a_0 t^{4n-4} + a_1 t^{2n-2} + a_2}{t^{2n-1}(a_3 t^{2n-2} + a_4)}$
Full causal theory ($\varepsilon = 1$)	$\xi = \frac{a_5 t^{6n-6} + a_6 t^{4n-4} + a_7 t^{2n-2} + a_8}{t^{2n-1}(a_9 t^{4n-4} + a_{10} t^{2n-2} + a_{11})}$

$$\Pi = -\left[\frac{m_0}{t^2} + \frac{m_1}{t^{2n}}\right], \tag{55}$$

$$m_0 = \frac{(1+\gamma)n^2 - 2n}{24\pi K} \quad \text{and}$$

$$m_1 = \frac{1}{8\pi K}\left(\frac{(c_1^2 + c_2^2 + c_3^2) + \gamma(c_1 c_2 + c_2 c_3 + c_1 c_3)}{9V_0^2} + \frac{3(1-\gamma)\beta_0^2}{4}\right).$$

In this case the relations between bulk viscosity coefficient and cosmic time t are in Table 1. Here constants a_i $(i = 0, 1, \ldots, 11)$ are constants expressed in terms of c_1, c_2, c_3, β_0, n, K, V_0 and γ. The exact forms of constants a_i are not mentioned due to lengthy expressions. In all cases bulk viscosity is decreasing with evolution of the universe.

Case II: Power Law relation between displacement field and Hubble parameter

In this case we assume the form of the displacement vector field $\beta = \beta_0 H^2$ as in case II of previous section.

Again, considering equation of state (50), we get following solutions for energy density and bulk viscous stress

$$\rho = \left(\frac{k_2}{(at+b)^2} + \frac{k_3}{(at+b)^4}\right), \tag{56}$$

$$\Pi = -\left[\frac{m_2}{(at+b)^2} + \frac{m_3}{(at+b)^4}\right], \tag{57}$$

where

$$m_2 = \frac{\gamma a^2}{6\pi K} \quad \text{and}$$

$$m_3 = \frac{[12(c_1^2 + c_2^2 + c_1 c_2) + \beta_0^2 a^4] + \gamma[12(c_1 c_2 + c_2 c_3 + c_1 c_3) - \beta_0^2 a^4]}{54\pi K}.$$

The expression for bulk viscosity coefficient in terms of cosmic time t are given in Table 2. Here constants b_i $(i = 0, 1, \ldots, 11)$ are constants expressed in terms of c_1, c_2, c_3, β_0, n, K, V_0 and γ. The exact forms of constants b_i are also not mentioned due to lengthy expressions. Further it can be seen that bulk viscosity is decreasing with evolution of the universe.

Table 2 Forms of bulk viscosity coefficient when $\beta=\beta_0 H^2$

Non causal theory ($\tau=0$)	$\xi=[\frac{m_2}{2a(at+b)}+\frac{m_3}{2a(at+b)^3}]$
Truncated causal theory ($\varepsilon=0$)	$\xi=\frac{b_0(at+b)^4+b_1(at+b)^2+b_2}{(at+b)^3[b_3(at+b)^2+b_4]}$
Full causal theory ($\varepsilon=1$)	$\xi=\frac{b_5(at+b)^6+b_6(at+b)^4+b_7(at+b)^2+b_8}{(at+b)^3[b_9(at+b)^4+b_{10}(at+b)^2+b_{11}]}$

4.2 Cosmological Models with Variable Gravitational Constant

In this section, assuming the widely considered power law relation between gravitational constant and cosmic time t as

$$G=G_0 t^m. \tag{58}$$

And negative constant deceleration parameter favored by present day observations, we will study the behavior of other parameters of the model. Let

$$q=\alpha \quad (\alpha>0). \tag{59}$$

From (14) and (59), we get

$$V=V_0 t^{3/1-\alpha}. \tag{60}$$

Equations (21) and (60) are similar and substituting the value of constant $\frac{3}{1-\alpha}=n$, we get solutions (22)–(24).

The value of displacement field, energy density, pressure and bulk viscosity can be obtained using (22)–(24), (43)–(46), (49) and (50) as

$$\beta^2=\frac{u}{t^2}+\frac{v}{t^{2n}}+\frac{w}{t^{2n-m}}, \tag{61}$$

$$\rho=\frac{k_4}{t^{m+2}}-\frac{k_5}{t^{2n}}, \tag{62}$$

$$\Pi=-\left(\frac{m_4}{t^{m+2}}+\frac{m_5}{t^{2n+m}}+\frac{m_6}{t^{2n}}\right), \tag{63}$$

where

$$u=\frac{-4mn^2}{9(2n-m-2)}, \qquad v=\frac{4(c_1c_2+c_2c_3+c_1c_3)}{27V_0^2},$$

$$k_4=\frac{1}{8\pi G_0}\left(\frac{n^2}{3}+\frac{mn^2}{3(2n-m-2)}\right), \qquad k_5=\frac{3w}{32\pi G_0},$$

$$m_4=\frac{1}{8\pi G_0}\left(\frac{(1+\gamma)n^2-2n}{3}-\frac{(1-\gamma)mn^2}{3(2n-m-2)}\right),$$

$$m_5=\frac{c_1^2+c_2^2+2c_1c_2+c_2c_3+c_1c_3}{72\pi G_0 V_0^2},$$

$$m_6=\frac{(1-\gamma)3w}{32\pi G_0} \quad \text{and} \quad w \text{ is constant of integration.}$$

Table 3 Forms of bulk viscosity coefficient when $G = G_0 t^m$

Non causal theory ($\tau = 0$)	$\xi = (\frac{m_4}{nt^{m+1}} + \frac{m_5}{nt^{2n+m-1}} + \frac{m_6}{nt^{2n-1}})$
Truncated causal theory ($\varepsilon = 0$)	$\xi = \frac{d_0 t^{4n-2m-4} + d_1 t^{2n-2m-2} + d_2 t^{2n-m-2} + d_3 t^{-m} + d_4}{t^{2n-1}(d_5 t^{2n-m-2} + d_6 t^{-m} + d_7)}$
Full causal theory ($\varepsilon = 1$)	$\xi = \frac{d_8 t^{6n-3m-6} + d_9 t^{4n-3m-4} + d_{10} t^{4n-2m-4} + d_{11} t^{2n-2m-2} + d_{12} t^{2n-m-2} + d_{13} t^{-m} + d_{14}}{t^{2n-1}(d_{15} t^{4n-2m-4} + d_{16} t^{2n-2m-2} + d_{17} t^{2n-m-2} + d_{18} t^{-m} + d_{19})}$

Further, in this case the relations between bulk viscosity coefficient and cosmic time t are in Table 3. Here constants d_i ($i = 0, 1, \ldots, 19$) are constants expressed in terms of c_1, c_2, c_3, β_0, n, K, V_0 and γ. The exact forms of constants d_i are not mentioned due to lengthy expressions. Further it can be seen from aforesaid table that bulk viscosity is decreasing with evolution of the universe.

5 Conclusion

In this paper we have studied Bianchi Type-I cosmological model with uniform and variable gravitational constant in the presence and absence of bulk viscosity. This paper is divided mainly in two parts. In the first part of the paper we have considered uniform gravitational constant. Motivated by power law relation between cosmological constant and cosmic time t, we have also considered a similar relation between displacement vector field β and cosmic time t. The assumption $\beta = \beta_0 t^{-n}$ provided us a very interesting cosmological model which shows anisotropic behavior in the early stages of the universe. The cosmological model approaches to isotropic behavior with evolution of the universe. This is in favour of observational results. It can be easily seen that for all $n > 3$ (22)–(24) present accelerated expanding model of the universe. However in other case $\beta \propto H^2$ the value of deceleration parameter is positive ($q = 1/2$). Though present day observations and literature favour accelerating model of the universe, the decelerating model should not be simply ruled out as our knowledge and instruments to measure various parameter of the universe are not complete and perfect. In the literature there are examples of some results which were in and out of scientific consideration.

The second part of the paper presents study of cosmological models with variable gravitational constant. Unlike the first part, we assumed negative deceleration constant and the relation $G = G_0 t^m$ for studying the behavior of cosmological parameters.

The observational upper bound on $\frac{\dot{G}}{G} < (1.10 \pm 1.07) \times 10^{-11}\ \text{yr}^{-1}$ (refer [51] and references there in) puts limit on $m \leq (0.11 \pm 0.107)$.

In all cases energy density, pressure, bulk viscosity and temperature are decreasing with evolution of the universe in fare agreement with observations.

Acknowledgement Authors would like to thank Inter-University Centre for Astronomy and Astrophysics Pune for providing facilities where part of this work was completed.

References

1. Weyl, H.: Sitzungsberichte der Preussischen Academie der Wissenschaften, p. 465 (1918)

2. Lyra, G.: Math. Z. **54**, 52 (1951)
3. Sen, D.K.: Z. Phys. **149**, 311 (1957)
4. Halford, W.D.: Aust. J. Phys. **23**, 863 (1970)
5. Sen, D.K., Dunn, K.A.: J. Math. Phys. **12**, 578 (1971)
6. Halford, W.D.: J. Math. Phys. **13**, 1399 (1972)
7. Bhamara, K.S.: Aust. J. Phys. **27**, 541 (1974)
8. Karade, T.M., Borikar, S.M.: Gen. Relativ. Gravit. **9**, 431 (1978)
9. Reddy, D.R.K., Innaiah, P.: Astrophys. Space Sci. **136**, 191 (1987)
10. Soleng, H.H.: Gen. Relativ. Gravit. **19**, 1213 (1987)
11. Beesham, A.: Aust. J. Phys. **41**, 833 (1988)
12. Singh, T., Singh, G.P.: J. Math. Phys. **32**, 2456 (1991)
13. Singh, T., Singh, G.P.: Il. Nuovo Cimento B **106**, 617 (1991)
14. Singh, T., Singh, G.P.: Astrophys. Space Sci. **182**, 189 (1991)
15. Singh, T., Singh, G.P.: Int. J. Theor. Phys. **31**, 1433 (1992)
16. Singh, T., Singh, G.P.: Fortschr. Phys. **41**, 737 (1993)
17. Singh, G.P., Desikan, K.: Pramana **49**, 205 (1997)
18. Rahaman, F., Chakraborty, S., Das, S., Mukherjee, R., Hossain, M., Begum, N.: Astrophys. Space Sci. **288**, 483 (2003)
19. Rahaman, F., Begum, N., Bag, G., Bhui, B.C.: Astrophys. Space Sci. **299**, 211 (2005)
20. Bali, R., Chandnani, N.K.: J. Math. Phys. **49**, 032502 (2008)
21. Bali, R., Chandnani, N.K.: Astrophys. Space Sci. **318**, 225 (2008)
22. Singh, J.K.: Astrophys. Space Sci. **314**, 361 (2008)
23. Rahaman, F., Ghosh, P.: Astrophys. Space Sci. **317**, 127 (2008)
24. Pradhan, A., Kumar, S.S.: Astrophys. Space Sci. **321**, 137 (2009)
25. Pradhan, A.: Commun. Theor. Phys. **51**, 378 (2009)
26. Bali, R., Chandnani, N.K.: Int. J. Theor. Phys. **48**, 1523 (2009)
27. Gron, O.: Astrophys. Space Sci. **173**, 191 (1990)
28. Maartens, R.: Class. Quantum Gravity **12**, 1455 (1995)
29. Mak, M.K., Harko, T.: Gen. Relativ. Gravit. **30**, 800 (1998)
30. Singh, G.P., Beesham, A.: Aust. J. Phys. **52**, 1039 (1999)
31. Pradhan, A., Pandey, V.K.: Int. J. Mod. Phys. D **11**, 893 (2002)
32. Mak, M.K., Harko, T.: Int. J. Mod. Phys. D **11**, 447 (2002)
33. Wang, X.X.: Astrophys. Space. Sci. **93**, 433 (2004)
34. Yadav, M.K., Roy, A., Pradhan, A.: Int J. Theor. Phys. **46**, 2677 (2007)
35. Calistete, R., Jr., Fabris, J.C., Tossa, J., Zimdhal, W.: Phys. Rev. D **76**, 103516 (2007)
36. Singh, C.P., Kumar, S.: Int. J. Theor. Phys. **48**, 925 (2008)
37. Sahni, A., Starobinsky, A.: Int. J. Mod. Phys. D **9**, 373 (2000)
38. Beesham, A.: Phys. Rev. D **48**, 3539 (1993)
39. Kalligas, D., Wesson, P., Everitt, C.W.F.: Gen. Relativ. Gravit. **24**, 351 (1992)
40. Kalligas, D., Wesson, P., Everitt, C.W.F.: Gen. Relativ. Gravit. **27**, 645 (1992)
41. Lima, J.A.S., Carvalho, J.C.: Gen. Relativ. Gravit. **26**, 909 (1995)
42. Arbab, A.I.: Gen. Relativ. Gravit. **29**, 61 (1997)
43. Ferreira, P.G., Joyce, M.: Phys. Rev. Lett. **79**, 4740 (1997)
44. Copeland, E.J., Liddle, A.R., Wands, D.: Phys. Rev. D **57**, 4686 (1998)
45. Vishwakarma, R.G.: Mon. Not. R. Astron. Soc. **345**, 545 (2003)
46. Vishwakarma, R.G., Narlikar, J.V.: J. Astrophys. Astron. **28**, 17 (2007)
47. Padmanabham, T., Chitre, S.M.: Phys. Lett. A **120**, 433 (1987)
48. Singh, J.P., Pradhan, A., Singh, A.K.: Astrophys. Space Sci. **314**, 83 (2008)
49. Bali, R., Singh, J.P.: Int. J. Theor. Phys. **47**, 3288 (2008)
50. Maartens, R.: Extended irreversible thermodynamics in relativity. In: Maharaj, S. (ed.) Hanno Rund Lecture Notes at University of Natal, Durban, South Africa (18 Sept. 1996). arXiv:astro-ph/9609119v1
51. Uzan, J.-P.: Rev. Mod. Phys. **75**, 403 (2003)